# Slope Fields – How to Make One

Previously we talked about how in some instances we can't actually find the antiderivative by the methods we know. If we want to find something out about the shape of the original function anyway (when we are given the derivative) we can construct a slope field. You will need to do this on the AP exam and on our test.

Let's take the differential equation $dy/dx = x^2$. We, of course, already know how to solve this. We would get $y = (1/3)x^3 + C$. Without initial conditions we would not be able to get an exact answer, but we know that $y = (1/3)x^3 + C$ describes the family of curves whose derivatives are $x^2$. If we were struck with amnesia and not able to remember how to take the antiderivative, we could construct a slope field.

To do this, **make a table** of sample x-values and slopes:

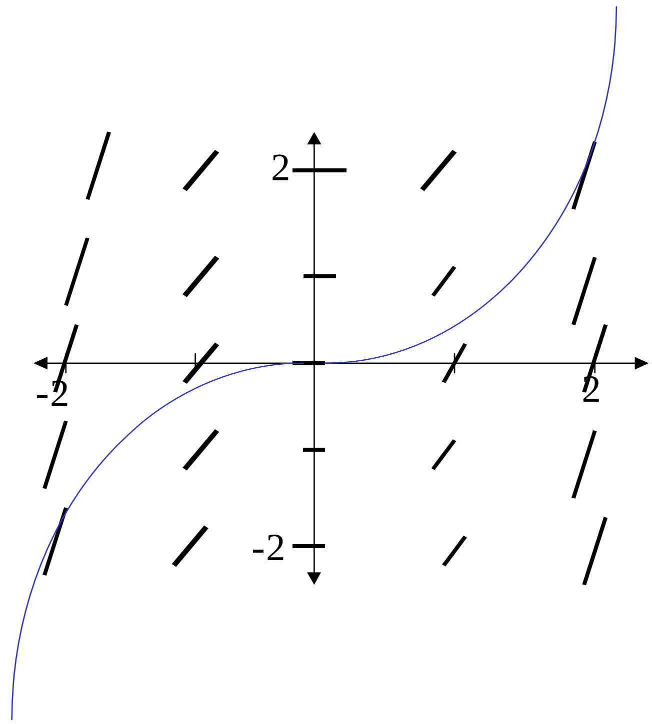
| x-value | slope ($dy/dx = x^2$ ) |
|---|---|
| 2 | 4 |
| 1 | 1 |
| 0 | 0 |
| -1 | 1 |
| -2 | 4 |

**The x-values** I just choose randomly and it does not really matter what values you use. However, that said, be smart about it and keep your values rather close to the origin and easy to work with. On a test or on the AP you will be told what values to use.

**The slopes** I find by putting those x-values into our slope equation, into the derivative.

Now I will draw this on an xy-plane. At each x-value within a 4 X 4 grid, I will draw a short tangent line whose slope corresponds to the slope in our chart. Since there is no y-value in our differential equation, the slope will be the same no matter what y-value I have paired with the x and I can just repeat the same tangent segment for all values of y at a given value of x.

This is what our slope field will look like:




For each x-value there is a short line representing the slope at that x-value. If you follow the slopes through the field, we can see a curve (which I have drawn in blue) - actually a family of curves, because we do not know what the vertical shift is.

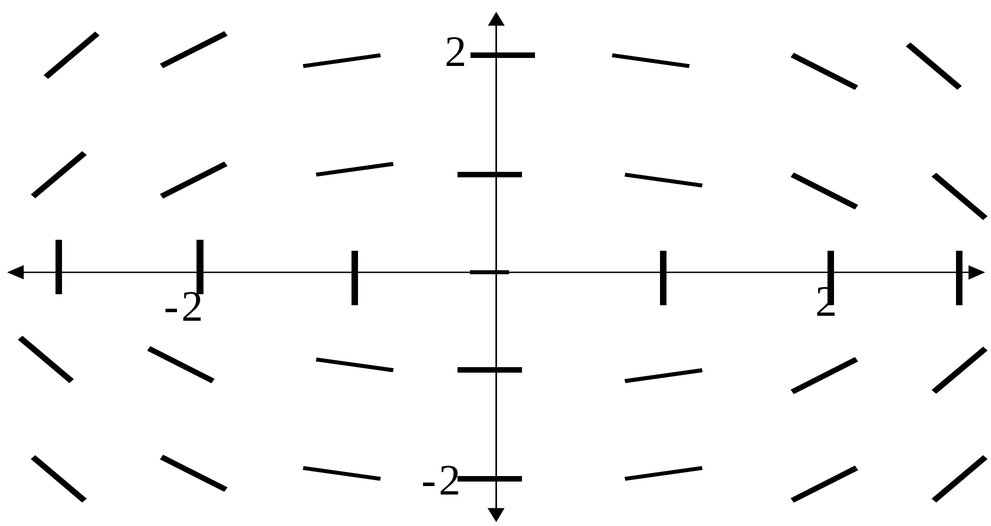
***** note** – the slope field is composed of only the short segments that represent the slope. **Do not draw in the curve unless you are asked to!** I just did it here so that you could see that the function $y = (1/3)x^3 + C$ does follow the slopes we found from our table and plotted on the graph.

Let's try another one. Let $\frac{dy}{dx} = \frac{-4x}{9y}$. When we make the table this time, we will have to deal with the fact that the y is part of the slope equation. Since I have no way of knowing what the y-value is for any one x-value, I will have to just look into the two situations that I <u>can</u> determine: when y is negative and when it is positive. Here is the table:

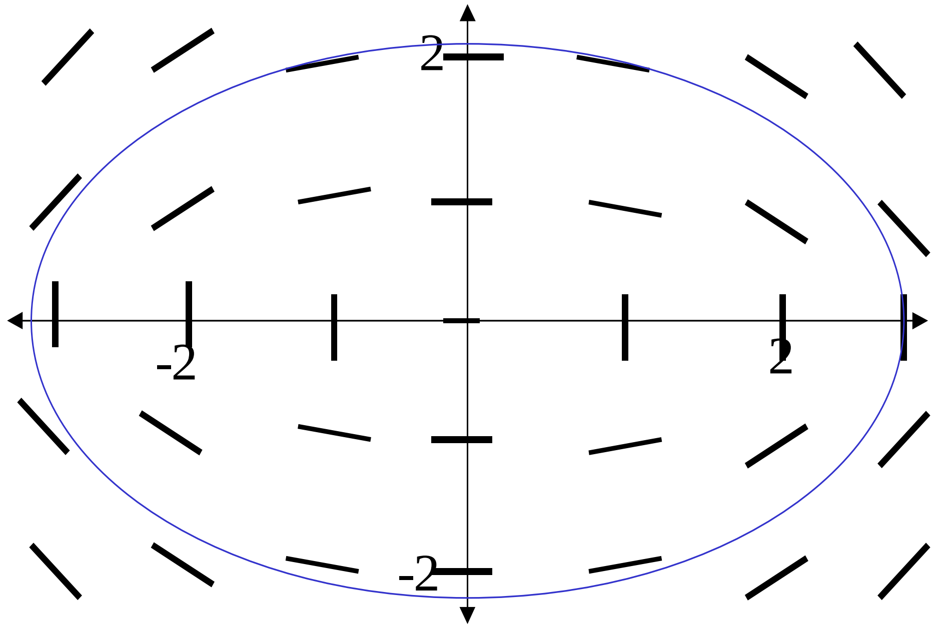
| x-value | y-value | slope $\left(\frac{dy}{dx}=\frac{-4x}{9y}\right)$ |
|---|---|---|
| 3 | + | -4/3 |
| 2 | + | -8/9 |
| 1 | + | -4/9 |
| -1 | + | 4/9 |
| -2 | + | 8/9 |
| -3 | + | 4/3 |
| 0 | + or - | 0 |
| 3 | - | 4/3 |
| 2 | - | 8/9 |
| 1 | - | 4/9 |
| -1 | - | -4/9 |
| -2 | - | -8/9 |
| -3 | - | -4/3 |
| along x-axis | 0 | no slope |

I chose random x-values.
Then I knew that y could be positive or negative or equal to zero.
Then I put that information into my derivative to determine what the slope might look like.

Now, translate this data to an xy-plane:




You can visualize what the curve will be, an ellipse, with center at the origin. Again, we don't know for sure where it is because we do not have an initial value. We can, however, draw in an estimated shape:




By the way, our slope field was fairly accurate. Our differential equation was found by taking the derivative of the ellipse with equation $4x^2 + 9y^2 = 36$, so it matches the slope field.

On the AP exam (and our test), you will either be asked to draw the slope field given the differential equation, or you may be given a slope field and asked to match it with a possible function's equation.